An Automatic Mass-Trim System for Spinning Spacecraft

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Long-term precision pointing of the spin axis of a spinning satellite may be impossible because attitude control systems cannot counteract the effects of sensor-vehicle misalignments, the motion of the principle axes of inertia, and body-fixed disturbing torques. An automatic mass-trim system (AMTS) consisting of two pairs of movable trim masses is proposed as a means of providing control torques to eliminate errors resulting from these sources. An analysis of the equations of motion of a rigid spinning body with internal moving masses relates the dynamics of mass motion to the attitude motion of the spinning body. Specializing the geometry to a two-pair system allows calculation of the control torque available and the range through which the principal axis set may be moved relative to body-fixed coordinates. A control law for an automatic system used in conjunction with a previously developed spinning-vehicle attitude controller is presented and analyzed using root locus techniques. An analog simulation of the vehicle with trim system verifies the design.

Nomenclature

a = time constant of AMTS spin-frequency filter
$\vec{a}, a_{x,y,z}$ = eigenvector of moment of inertia tensor **I**
E = unit tensor
$\vec{H}$ = system angular momentum
$\vec{H}_m$ = angular momentum of internal moving masses
I = moment of inertia tensor
I_{ij} = $i, j = x, y, z$ elements of **I** (moments and products of inertia)
I_s = vehicle spin axis (maximum) moment of inertia
I_t = vehicle transverse (intermediate) moment of inertia
K_M = acceleration gain for AMTS loop
K_p, K_v = attitude control loop feedback gains
$\vec{M}, M_{x,y}$ = moment
$\vec{M}_C, M_{Cx,y}$ = control moment
$\vec{M}_{DB}, M_{DBx,y}$ = disturbing moments fixed in the body
$\vec{M}_{DI}, M_{DIx,y}$ = disturbing moments fixed in inertial space
m = moment of inertia ratio, $(I_s - I_t)/I_t$
m_i = mass of ith particle
n = moment of inertia ratio, I_s/I_t
$\vec{Q}, \vec{Q}_{x,y}$ = ratios $\vec{M}/I_t$
$\vec{Q}_C, Q_{Cx,y}$ = ratio $\vec{M}_C/I_t$
$\vec{Q}_D, \vec{Q}_{DB}, \vec{Q}_{DI}$ = ratios $\vec{M}_D/I_t, \vec{M}_{DB}/I_t, \vec{M}_{DI}/I_t$
Q_T = $2\mu R\beta^2/I_t$
R = distance of movable mass in $X_B - Y_B$ plane from vehicle symmetry axis
$\vec{r}, r_{x,y,z}$ = radius vector
v_i = velocity of particle i
X_B, Y_B, Z_B = axes of an orthogonal coordinate system fixed in the body
X_I, Y_I, Z_I = axes of an orthogonal coordinate system fixed in inertial space
x_i, y_i, z_i = position coordinates of particle i
$Z_{x,y}, Z_{x^+}, Z_{x^-}, Z_{y^+}, Z_{y^-}$ = vertical distance of the $\pm x$ and $\pm y$ AMTS trim masses from the $X_B - Y_B$ plane
α = phase angle
β = spin angular velocity
$\gamma_{x,y}$ = star-tracker attitude angles for a spinning vehicle
μ = mass of one AMTS trim mass
σ = variable of integration
ϕ = $\cos^{-1} a_z/|\vec{a}|$
$\omega_{x,y,z}$ = angular velocity components along body x, y, z axes
$d^I(\ )/dt$ = time derivative of a vector in inertial frame
$d^B(\ )/dt$ = time derivative of a vector in body frame
$\vec{\omega}^{A-B}$ = angular velocity of frame A with respect to frame B
$(\dot{\ })$ = time derivative of a quantity
$A * B$ = A convolved with B

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Introduction

A COMMON requirement for many spinning satellite designs is to control the spin axis with an accuracy of the order of seconds of arc. Satellites scanning the Earth's surface for resources or weather pictures, and communications satellites aiming high-gain, narrow beam-width antennas are two systems requiring this accuracy. Passive control of nutation with energy dissipating "wobble dampers" and active control using magnetic torquing, reaction jets, momentum wheels, or control moment gyros have all been utilized with varying degrees of success.

For an active system, the accuracy of the pointing control depends directly on knowledge of the relative location of the principal axis about which the body is spinning and the sensor that is producing the pointing error signal. If the principal axis set is moving in the body (e.g., due to expulsion of control gas), or if the sensor axis and nominal spin axis are not correctly aligned, a pointing error arises that may be impossible to eliminate. In addition, moments fixed in the satellite coordinates (e.g., a leaking control jet) have a first-order effect identical to sensor-principal axis misalignments and hence produce similar attitude errors.

Thus, if precise control over a long period of time is required, it may be necessary to trim the location of the principal axis set automatically to align the sensor and spin axes, or create body-fixed control moments that counteract disturbance torques. This paper presents one method for accomplishing this which consists of moving pairs of trim masses inside the satellite body. A more detailed explanation is contained in a recent Stanford University report.[1]

A number of authors have investigated the effects of internal moving masses on the dynamics of spinning bodies. Geophysicists have long been interested in the subject because the Earth's spin axis is affected by motion of the crust. Munk and MacDonald[2] give a treatment of the problem in this context. An early paper by Roberson[3] derives the torques on a satellite caused by internal moving parts and calculates the effects caused by point masses translating and vibrating along lines through the mass center. Kane,[4] in the first of a two-paper series, describes the stability criteria for an axisymmetric spinning body with an internal particle oscillating along a principal axis of inertia. In the second paper, Kane and Sobala[5] examine a method of stabilizing a spinning symmetric satellite with a pair of oscillating point masses. Stabilization is obtained by having the pair oscillate in a prescribed manner along the symmetry axis. Michelini et al.[6] analyzed the sloshing of fuel in spinning satellites. They found a quasi-steady-state solution and conditions whereby oscillations can be induced in a body with no initial angular rates.

Experimental work in this area is rather limited. Kurzhals and Adams[7] built simulators to investigate the relationship between controlled behavior and vehicle inertia ratios of spinning symmetric space stations. They included a small oscillating spring and mass to simulate a man walking inside the station. Adams[8,9] used manually adjustable masses to change the inertia ratio and create products of inertia for a simulator he used to test various control mechanizations.

This paper describes the analysis and design of a moving mass-trim system for a symmetric spinning satellite. The equations of motion of a spinning rigid body with internal moving masses are developed and presented in a form that clearly illustrates the separate contributions of trim-mass motion and position. A specific geometry is proposed for the trim masses, allowing expressions for torques applied to the body by these masses to be obtained from the general equations.

An attitude controller for a symmetric spinning satellite proposed by Lange et al.[10] has been shown to have steady-state errors proportional to attitude sensor-vehicle spin axis misalignments and moments fixed in the body. To illustrate this approach, an automatic mass-trim system that compensates for these error sources is synthesized for such a satellite and an analysis of important control parameters is carried out employing the results of the first section. An analog computer simulation of the combined satellite-trim system has been used to verify the design.

Equations of Motion

In this section the equations of motion of a rigid body with internal moving mass are developed. Assumptions made regarding the specific geometry of a satellite with a moving mass-trim system make it possible to relate mass position and velocity to torques acting on the body and to the motion of the principal axis set.

If $\vec{H}$ is the total angular momentum of the satellite with moving internal masses, then

$$\vec{H} = \mathbf{I}\cdot\vec{\omega}^{B-I}+\vec{H}_m \tag{1}$$

where

$$\vec{H}_m \triangleq \sum_{i=1}^{n} \vec{r}_i \times m_i(d^B\vec{r}_i/dt)$$

is the additional angular momentum due to the motion of mass relative to some body-fixed coordinate system. By Coreolis' law,

$$d^I\vec{H}/dt = (d^B\mathbf{I}/dt)\cdot\vec{\omega}^{B-I}+\mathbf{I}\cdot d^B\vec{\omega}^{B-I}/dt+\vec{\omega}^{B-I}\times\mathbf{I}\cdot\vec{\omega}^{B-I}+d^B\vec{H}_m/dt+\vec{\omega}^{B-I}\times\vec{H}_m \tag{2}$$

Thus, additional terms arise that are not found in the corresponding rigid-body equation.

Expansion of Eq. (2) will indicate its relationship to the rigid-body equations

$$\begin{bmatrix} M_{Bx}-M_{PIx}-M_{MDx} \\ M_{By}-M_{PIy}-M_{MDy} \\ M_{Bz}-M_{PIz}-M_{MDz} \end{bmatrix} = \begin{bmatrix} I_{xx}\dot{\omega}_x-(I_{yy}-I_{zz})\omega_y\omega_z \\ I_{yy}\dot{\omega}_y-(I_{zz}-I_{xx})\omega_z\omega_x \\ I_{zz}\dot{\omega}_z-(I_{xx}-I_{yy})\omega_x\omega_y \end{bmatrix} \tag{3}$$

where the M_{PI} and M_{MD} terms, defined below, represent the contributions due to products of inertia and the dynamics of mass motion, respectively.

Products of inertia contribute the following terms:

$$\begin{bmatrix} M_{PIx} \\ M_{PIy} \\ M_{PIz} \end{bmatrix} =$$

$$\begin{bmatrix} I_{xy}\dot{\omega}_y+I_{xz}\dot{\omega}_z+(I_{zx}\omega_x+I_{zy}\omega_y)\omega_y-(I_{yx}\omega_x+I_{yz}\omega_z)\omega_z \\ I_{yx}\dot{\omega}_x+I_{yz}\dot{\omega}_z+(I_{xy}\omega_y+I_{xz}\omega_z)\omega_z-(I_{zx}\omega_x+I_{zy}\omega_y)\omega_y \\ I_{zx}\dot{\omega}_x+I_{zy}\dot{\omega}_y+(I_{yx}\omega_x+I_{yz}\omega_z)\omega_x-(I_{xy}\omega_y+I_{xz}\omega_z)\omega_y \end{bmatrix} \tag{4}$$

The dynamics of mass motion produce

$$\begin{bmatrix} M_{MDx} \\ M_{MDy} \\ M_{MDz} \end{bmatrix} = \begin{bmatrix} \dot{I}_{xx}\omega_x+\dot{I}_{xy}\omega_y+\dot{I}_{xz}\omega_z+\dot{H}_{mx}+H_{mz}\omega_y-H_{my}\omega_z \\ \dot{I}_{xy}\omega_x+\dot{I}_{yy}\omega_y+\dot{I}_{yz}\omega_z+\dot{H}_{my}+H_{mx}\omega_z-H_{mz}\omega_x \\ \dot{I}_{xz}\omega_x+\dot{I}_{yz}\omega_y+\dot{I}_{zz}\omega_z+\dot{H}_{mz}+H_{my}\omega_x-H_{mx}\omega_y \end{bmatrix}$$

The right side of Eq. (3) is Euler's equations for a rigid spinning body, and the left side may be thought of as a forcing function arising from the nonrigid property.

By making a set of assumptions regarding the inertial properties of the vehicle and the geometry of the moving masses, Eq. (3) can be simplified considerably. Thus, for the purposes of the following analysis, the satellite (rigid-body portion) is assumed to be spinning about its axis of inertial symmetry such that ω_x/β, ω_y/β, $\dot{\omega}_x/\beta$, $\dot{\omega}_y/\beta$ are first-order small. The four trim system masses are located on the intermediate axes of inertia around the vehicle circumference with lines of travel parallel to the nominal (maximum inertia) spin axis. This mass is such that I_{yz}/I_t and I_{xz}/I_t are small to first order.

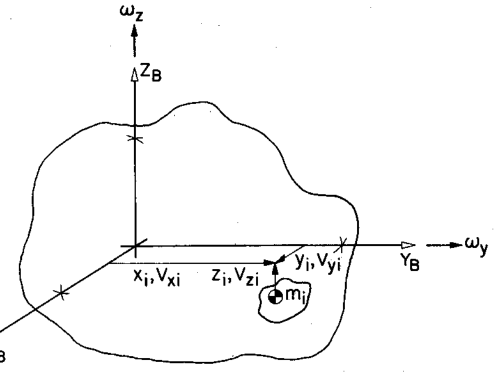

Fig. 1 Body-fixed coordinate system and coordinates for an internal moving mass.

Before applying these assumptions, $\vec{H}_m$ is first expanded in body coordinates. Referring to Fig. 1,

$$\begin{bmatrix} H_{mx} \\ H_{my} \\ H_{mz} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} m_i(r_{yi}v_{zi}-r_{zi}v_{yi}) \\ \sum_{i=1}^{n} m_i(r_{zi}v_{xi}-r_{xi}v_{zi}) \\ \sum_{i=1}^{n} m_i(r_{xi}v_{yi}-r_{yi}v_{xi}) \end{bmatrix}$$

Thus, (see Fig. 2)

$$\begin{bmatrix} H_{mx} \\ H_{my} \\ H_{mz} \end{bmatrix} = \begin{bmatrix} \mu R(\dot{Z}_{y^+}-\dot{Z}_{y^-}) \\ -\mu R(\dot{Z}_{x^+}-\dot{Z}_{x^-}) \\ 0 \end{bmatrix}$$

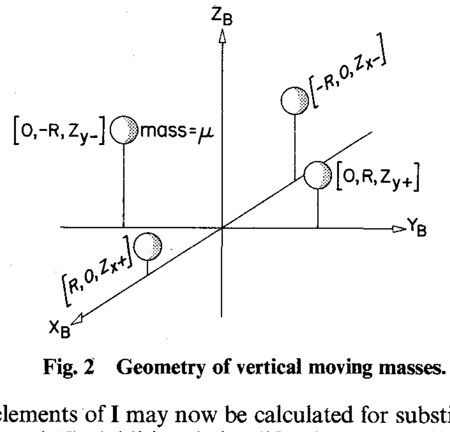

Fig. 2 Geometry of vertical moving masses.

The elements of I may now be calculated for substitution into Eqs. (4) and (5). Additional simplification results if we require the system center of mass to remain stationary in vehicle coordinates during trim-mass motion. Then,

$$Z_{x,y^-} = -Z_{x,y^+} \triangleq -Z_{x,y}$$
$$\dot{Z}_{x,y^-} = -\dot{Z}_{x,y^+} \triangleq -\dot{Z}_{x,y}$$

Equations (4) and (5) become, to first-order,

$$\begin{bmatrix} M_{PIx} \\ M_{PIy} \\ M_{PIz} \end{bmatrix} \doteq \begin{bmatrix} 2\mu RZ_y\beta^2 \\ -2\mu RZ_x\beta^2 \\ 0 \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} M_{MDx} \\ M_{MDy} \\ M_{MDz} \end{bmatrix} \doteq \begin{bmatrix} 4\mu(\dot{Z}_x Z_x + \dot{Z}_y Z_y)\omega_x + 2\mu R\ddot{Z}_y \\ 4\mu(\dot{Z}_x Z_x + \dot{Z}_y Z_y)\omega_y - 2\mu R\ddot{Z}_x \\ -4\mu R(\dot{Z}_x \omega_x + \dot{Z}_y \omega_y) \end{bmatrix} \tag{7}$$

Because of the low speed of the trim masses and the assumption that the ω_x, ω_y attitude motion is on the order of arc seconds/second, the $4\mu(\dot{Z}_x Z_x + \dot{Z}_y Z_y)\omega_{x,y}$ terms may be eliminated. As a result, the M_{MD} equations reduce to

$$\begin{bmatrix} M_{MDx} \\ M_{MDy} \\ M_{MDz} \end{bmatrix} \doteq \begin{bmatrix} 2\mu R\ddot{Z}_y \\ -2\mu R\ddot{Z}_x \\ 0 \end{bmatrix}$$

and the linearized form of Eq. (3) is

$$\begin{bmatrix} \dfrac{M_{Bx}}{I_t} - \dfrac{2\mu R Z_y \beta^2}{I_t} - \dfrac{2\mu R\ddot{Z}_y}{I_t} \\ \dfrac{M_{By}}{I_t} + \dfrac{2\mu R Z_x \beta^2}{I_t} + \dfrac{2\mu R\ddot{Z}_x}{I_t} \end{bmatrix} \doteq \begin{bmatrix} \dot{\omega}_x - \dfrac{I_t - I_s}{I_t}\beta\omega_y \\ \dot{\omega}_y - \dfrac{I_s - I_t}{I_t}\beta\omega_x \end{bmatrix} \tag{8}$$

where $M_{Bx,y}$ represent disturbing moments and in the case of a controlled body, the control torques as well.

Equations (8) represent the terms in Eq. (3) created by products of inertia and mass motion as torques on a rigid body. Another equally valid representation is that the mass motion inside the body has the effect of reorienting the principal axis set with respect to the original ($t = 0$) coordinate system. Because it is not possible to align the null axis of the attitude sensor with the axis of maximum inertia as precisely as is desired (less than 10^{-4} rad, for example) for many control problems, it is of great practical importance to be able to move the principal axes relative to the body.

To size a trim-system design correctly, knowledge of the range through which the principal axis set may be moved is useful. This angle, measured from the $t = 0$ principal axis set, can be calculated by rediagonalizing $\mathbf{I}$

$$\mathbf{I}_{(t>0)} = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

This is equivalent to solving for the orientation of the eigenvector $\vec{a}$ in the equation

$$\mathbf{I}\cdot\vec{a} = \lambda\mathbf{E}\cdot\vec{a} \tag{9}$$

Expanding Eq. (9) and substituting expressions for the components of $\mathbf{I}$ calculated previously, we arrive at a relationship for the orientation angles of the new principal axis set ϕ and θ (defined in Fig. 3)

$$\tan\phi \doteq \frac{4R\mu(Z_x^2 + Z_y^2)^{1/2}}{I_t} \times \left\{\left[m^2 + \frac{16R^2\mu^2}{I_t^2}(Z_x^2 + Z_y^2)\right]^{1/2} + m\right\}^{-1} \tag{10}$$

$$\tan\theta \doteq Z_y/Z_x, \qquad m \triangleq (I_s - I_t)/I_t$$

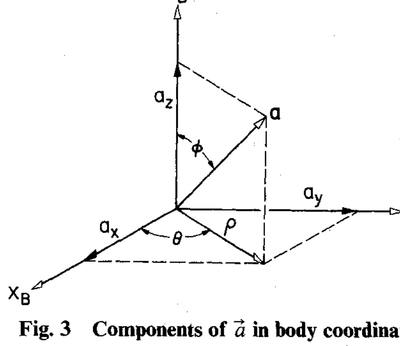

Fig. 3 Components of $\vec{a}$ in body coordinates.

A trim system consisting of four 3-kg masses on a 100-kg vehicle would have a typical ϕ_{max} (depending on geometry) of approximately 2.5°.

Control of an Automatic Mass-Trim System (AMTS)

The results obtained by Lange et al.[10] are presented first in this section to demonstrate the need for an AMTS. A linear control law is then proposed and control parameters selected based on a root locus analysis. The output of an analog simulation of a vehicle with an AMTS is presented.

In Ref. 10, a method of designing controllers for symmetric spinning spacecraft, called frequency symmetry, is developed. The authors design a typical spinning satellite controller by transforming Euler's equations for a rigid body spinning about its axis of symmetry with a constant rate β into star-tracker coordinates $\gamma_{x,y}$, and using their technique to select two control gains K_p and K_v.

Thus, for small angles,

$$\omega_x \doteq \dot{\gamma}_x - \beta\gamma_y, \qquad \omega_y \doteq \dot{\gamma}_y + \beta\gamma_x$$

and from Ref. 10, the equations of the controlled satellite, normalized as in Eq. (8), are

$$Q_{Dx} = \ddot{\gamma}_x + K_v\dot{\gamma}_x + (K_p + m\beta^2)\gamma_y - (2-n)\beta[\dot{\gamma}_y + (K_v/2)\gamma_y] \tag{11a}$$

$$Q_{Dy} = \ddot{\gamma}_y + K_v\dot{\gamma}_y + (K_p + m\beta^2)\gamma_y + (2-n)\beta[\dot{\gamma}_x + (K_v/2)\gamma_x] \tag{11b}$$

$$m \triangleq (I_s - I_t)/I_t, \qquad n \triangleq I_s/I_t \tag{11c}$$

Steady-state errors resulting from disturbing torques (Q_{DB} being body fixed, Q_{DI} inertially fixed) were found to be inversely proportional to K_p

$$\gamma_x(t\to\infty) \doteq Q_{DBx}/K_p + (Q_{DI}/K_p)\cos(\beta t + \alpha)$$

$$\gamma_y(t\to\infty) \doteq Q_{DBy}/K_p + (Q_{DI}/K_p)\sin(\beta t + \alpha)$$

The controller, therefore, can never completely eliminate errors caused by these disturbances. Q_{DB} also includes the first-order effects of products of inertia, as discussed earlier.

Figures 4 and 5 are analog simulations of the linear controller responding to initial conditions, with and without Q_{DB} and Q_{DI} acting on the body. The steady-state error is a circle with radius of length Q_{DI}/K_p whose center is displaced by Q_{DBx}/K_p and Q_{DBy}/K_p from the origin.

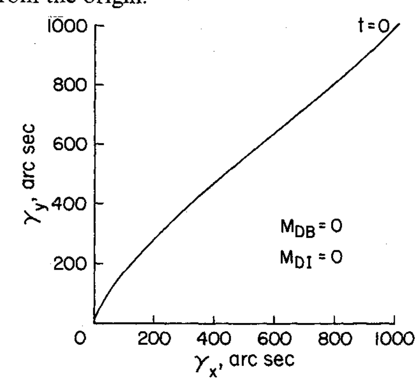

Fig. 4 Spinning-vehicle controller simulation without disturbing moments.

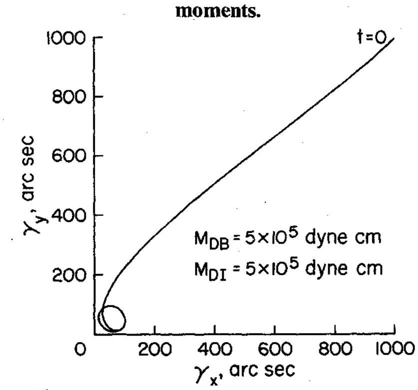

Fig. 5 Spinning-vehicle controller simulation with body-fixed and inertial disturbing moments.

By positioning movable masses inside the vehicle to create offsetting moments, as suggested by Eq. (6), the zero-frequency portion of the steady-state error, illustrated in Fig. 5, can be eliminated. A linear control law is proposed for such a system. Mass velocity (speed and direction) rather than mass position are controlled. The necessity to estimate the magnitude and direction of the apparent body-fixed moment is thereby eliminated, and knowledge of the position of each mass is also unnecessary.

Because the zero-frequency components of the steady-state error are directly proportional to $\vec{Q}_{DB}$, a control law was chosen in the form of

$$\dot{Z}_y = K_M \gamma_x{}^* e^{-at}, \qquad \dot{Z}_x = -K_M \gamma_y{}^* e^{-at} \tag{12}$$

where $Z_{x,y}$ refers to the notation in Eqs. (6). This results in an intergral controller, described in Ref. 10 as a method of eliminating steady-state errors. The control moment applied by the mass pair is

$$\begin{bmatrix} Q_{Mx} \\ Q_{My} \end{bmatrix} = \begin{bmatrix} \dfrac{2\mu R\beta^2 K_M}{I_t} \displaystyle\int_0^t \gamma_x(\sigma)^* e^{-a\sigma} d\sigma \\ \dfrac{2\mu R\beta^2 K_M}{I_t} \displaystyle\int_0^t \gamma_y(\sigma)^* e^{-a\sigma} d\sigma \end{bmatrix} \tag{13}$$

The e^{-at} term acts as a filter to separate the zero-frequency and spin-frequency components of γ.

The equations for the controlled body now become

$$\begin{bmatrix} Q_{Dx} \\ \\ \\ Q_{Dy} \\ \\ \end{bmatrix} = \begin{bmatrix} \ddot{\gamma}_x + \dot{\gamma}_x\left(K_v + \dfrac{Q_T K_M}{\beta^2} * e^{-at}\right) + \\ \gamma_x\left(K_p + m\beta^2 - \dfrac{aQ_T K_M}{\beta^2} * e^{-at}\right) + \\ Q_T K_M \displaystyle\int_0^t \gamma_x(\sigma) * e^{-a\sigma} d\sigma - (2-n)\beta\left(\dot{\gamma}_y + \dfrac{K_v}{2}\gamma_y\right) \\ \ddot{\gamma}_y + \dot{\gamma}_y\left(K_v - \dfrac{Q_T K_M}{\beta^2} * e^{-at}\right) + \\ \gamma_y\left(K_p + m\beta^2 + \dfrac{aQ_T K_M}{\beta^2} * e^{-at}\right) - \\ Q_T K_M \displaystyle\int_0^t \gamma_y(\sigma) * e^{-a\sigma} d\sigma + (2-n)\beta\left(\dot{\gamma}_x + \dfrac{K_v}{2}\gamma_x\right) \end{bmatrix} \tag{14}$$

where $Q_T = 2\mu R\beta^2/I_t$.

The trim-system gains K_M and a can be chosen based on a root locus analysis. Because a is the spin-frequency filter time constant, its value can be limited to $0 < a < \beta$. Stability and response of the system then can be determined by plotting a series of K_M root loci, each for a different value of a.

Selection of K_M is made once an appropriate a is chosen. To keep the trim system from tracking spin-frequency oscillations, a should be kept as small as possible. The root loci indicate that system stability and speed of response degrade as a is lowered; hence a tradeoff value must be accepted.

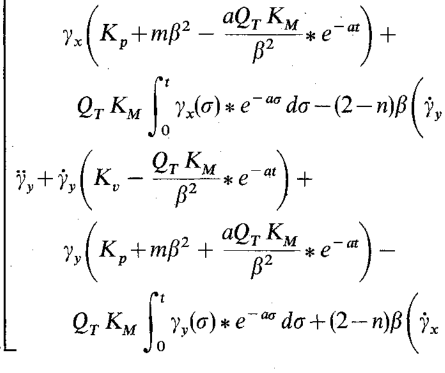

Fig. 6 Automatic mass-trim system response.

Figure 6 is an analog simulation of vehicle-trim system response for $a = 0.2$ sec^{-1} and $K_M = 37$ cm/sec^2/rad. Other system parameters are as follows:

$$K_p = 14.4 \text{ sec}^{-2}, \quad n = 1.804, \quad K_v = 10.0 \text{ sec}^{-1}$$
$$\beta = 1.0 \text{ rad/sec}, \quad m = 0.804, \quad Q_T = 1.981 \times 10^{-3} \text{ cm}^{-1} \text{ sec}^{-2}$$

Response to initial conditions and step inputs of inertial and body-fixed moments is satisfactory. The trim system oscillates only slightly, caused by the inertial torque. Under actual operating conditions, inertial torques are at least an order of magnitude less than those simulated here, making trim-system motion negligible.

The basic instability of a moving mass-trim system with the above control law is illustrated in Fig. 7, which is a root locus vs K_M with K_p and $K_v = 0$. This instability is caused by the $Q_T \ddot{Z}_{x,y}/\beta^2$ terms. Stability of the vehicle with an automatic mass-trim system, therefore, is dependent on the attitude control loop to damp these disturbing moments.

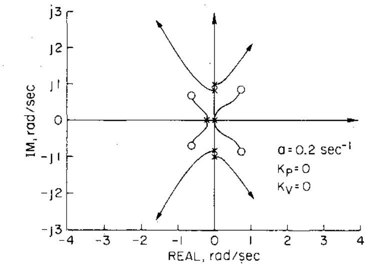

Fig. 7 Root locus of Eq. (14) vs K_M without attitude control loop. An AMTS, using the control law of Eq. (12), is unstable for $K_M > 0$ if there is no active attitude control.

Conclusions

Long-term precision pointing of the spin axis of a spinning satellite may be made difficult or impossible by the effects of disturbing moments, the changing inertial properties of the satellite caused by mass expelled from the vehicle, and misalignments between the attitude sensor null axis and the vehicle's principal axis set. An automatic mass-trim system, consisting of four movable masses located on the end of the vehicle's intermediate axes of inertia, can provide the control torques necessary to eliminate these error sources.

The system described in this paper is stable only with the use of an outer attitude control loop. When used in conjunction with the attitude controller, the trim-system response characteristics to inertial and body-fixed disturbances as demonstrated by analog simulation is quite satisfactory. Steady-state pointing errors on the order of several hundred arc seconds are zeroed (in the idealized simulation) automatically with a relatively short time delay and with a low sensitivity to non-zero-frequency disturbances.

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